Name Key

Review Questions

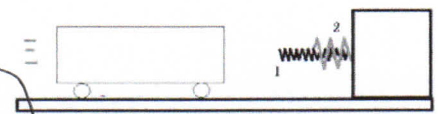
Energy / Work / Momentum

1. A 6,000 kg freight car rolls along rails with negligible friction. The car is brought to rest by a combination of two coiled springs (see image). Spring #1 has a spring constant of 1,600 N/m and spring #2 has a spring constant of 3,400 N/m. After the first spring compresses a distance of 30 cm, the second spring acts with the first to increase the force. The car comes to rest 50 cm after first contacting spring #1. Calculate the ~~work done~~ by each spring and the initial velocity of the freight car.

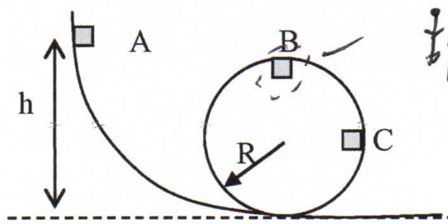
$E_{E(1)} = \frac{1}{2}(1600)(0.5)^2 = 200J$

$E_{E(2)} = \frac{1}{2}(3400)(0.2)^2 = 68J$

$268J = \frac{1}{2}(6000)(v^2)$ $v = 0.3\ m/s$



2. A small object of mass m slides without friction around a loop-the-loop apparatus as shown below. It starts from rest at point A at a height h above the bottom of the loop.



F_N is very small

$\frac{mv^2}{r} = mg + 0$

$v = \sqrt{Rg}$

$mgh = mg2R + \frac{1}{2}mRg$

$h = 2.5r$

a. What is the minimum value of h (in terms of R) such that the object moves around the loop without falling off at the top (point B)?

$h = 2.5R$

b. Assume that h = 3R. With what velocity will the object move at point C?

i. Get to simplest form

$mg3R = mgR + \frac{1}{2}mv^2$ $v = 2\sqrt{gR}$

3. A certain spring is found NOT to obey Hooke's law, but rather exerts a restoring force $F(x) = -40x - 9x^2$ if it is stretched or compressed a distance x. The units of the numerical factors are such that if x is in meters, then F will be in newtons.

a. Calculate the potential energy function U(x) for this spring. Let U=0 when x=0.

$F(x) = -\frac{U(x)}{dx}$ $U(x) = 20x^2 + 3x^3 + C$

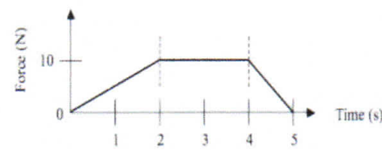
b. An object of mass 3 kg is attached to this spring, pulled a distance 1.2 m to the right on a frictionless horizontal surface and released. What is the velocity of the object when it is 0.5 m to the right of the x = 0 equilibrium position?

$U(1.2) = U(0.5) + \frac{1}{2}mv^2$

$v = 4.36\ m/s$

4. A 2.0 kg object experiences a force according to the graph above. If the object starts from rest:

a. What is the change in momentum of the object during the time interval?



Area is impulse

35 kg m/s

b. With what velocity will the object be traveling at the end of the time interval?

$mv_f - mv_i = 35$

$v_f =$ **17.5 m/s**

5. A 0.7 kg object accelerates according to the function a(t) = 4t – 2.1. If at a time of 2.0 seconds, the object is observed to be traveling at 5.0 m/s, with what momentum will the object travel with at a time of 3.0 seconds?

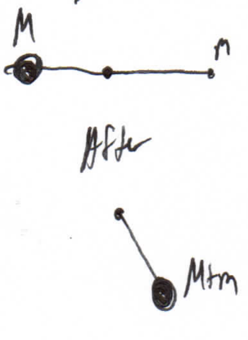
$\int a(t) = v(t) = 2t^2 - 2.1t + c$

$c = 1.2$ m/s

$v(3) = 12.9$

$p = (0.7)(12.9) =$ **9.03 kg m/s**

6. George of the jungle, with mass *m*, swings on a light vine hanging from a tree branch. A second vine of equal length hangs from the same point, and a gorilla of larger mass *M* swings in the opposite direction on it. Both vines are horizontal when the primates start from rest at the same moment. George and the gorilla meet at the lowest point of their swings. Each is afraid that one vine will break, so they grab each other and hang on. They swing upward together, reaching a point where the vines make an angle of 35.0 degrees with the vertical. Find the value of the ratio *m/M*.



Velocity of m before collision

$mgh = \frac{1}{2}mv^2$

$v_m = \sqrt{2gh}$

velocity of M

$v_M = \sqrt{2gh}$

Velocity of M+m

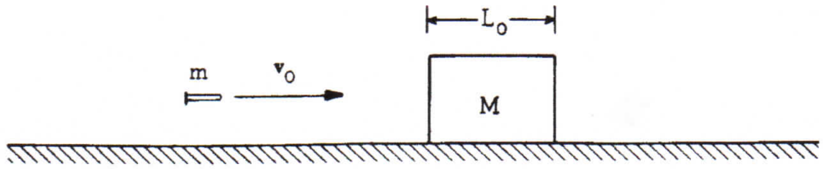
$\frac{1}{2}(M+m)v^2 = (M+m)gh$

$h = H - H\cos 35$

$v_{m+M} = \sqrt{2g(H - H\cos 35)}$

$mv_m + Mv_M = (m+M)v_{m+M}$

$$\frac{m}{M} = 0.4$$



Actual AP Free Response Problem (Time yourself for 15 minutes for best practice)

7. A bullet of mass m and velocity v_o is fired toward a block of thickness L_o and mass M. The block is initially at rest on a frictionless surface. The bullet emerges from the block with velocity $v_o/3$.

   a. Determine the final speed of block M.

$$mV_0 = m\frac{V_0}{3} + M V_f$$

$$V_f = \frac{2}{3} mV_0 \cdot \frac{1}{M} = \boxed{\frac{2mv_0}{3M}}$$

   b. If, instead, the block is held fixed and not allowed to slide, the bullet emerges from the block with aspeed $v_o/2$. Determine the loss of kinetic energy of the bullet

$$\frac{1}{2}mv^2 - \frac{1}{2}m\frac{v^2}{4} = \left(\frac{1}{2} - \frac{1}{8}\right)(mv^2) = \frac{3}{8}mv^2$$

   c. Assume that the retarding force that the block material exerts on the bullet is constant. In terms of L_o, what minimum thickness L should a fixed block of similar material have in order to stop the bullet?

$$FL_0 = \frac{1}{4}mv^2$$

$$F = \frac{mv^2}{4L_0}$$

$$\frac{mv^2}{4L_0} \cdot L = \frac{1}{2}mv^2$$

$$\boxed{L = 2L_0}$$

   d. When the block is held fixed, the bullet emerges from the block with a greater speed than when the block is free to move. Explain.

Energy is not transfered into the block.