Name KEY

Worksheet #2

Calculus and Relative Velocity

1. The speed of a bullet as it travels down the barrel of a rifle toward the opening is given by:

$$V = (-5.00 \times 10^7)t^2 + (3.00 \times 10^5)t$$

...where *v* is in meters per second and *t* is in seconds. The acceleration of the bullet just as it leaves the barrel is zero.

a. Determine the acceleration and position of the bullet as functions of time when the bullet is in the barrel.

$a(t) = \frac{dv}{dt} = (-10\times10^7)t + (3\times10^5)$

$x(t) = \int v = \left(-\frac{5}{3}\times10^7\right)t^3 + \left(\frac{3}{2}\times10^5\right)t^2$

b. Determine the time interval over which the bullet is accelerated.

0.003 s $(-10\times10^7)t + (3\times10^5) = 0 \quad t = 0.003$

c. Find the speed at which the bullet leaves the barrel.

450 m/s $v(0.003) = 450$ m/s

d. What is the length of the barrel?

0.90 m $x(0.003) = 0.9$m

2. A toy car has the velocity expression $v(t) = (1\ m/s^3)t^2 + 1\ m/s$. What will be this cars displacement from t = 0 sec to t = 2 sec?

4.67 m $\int_0^2 t^2 + 1\,dt = 4.67$ m/s

3. A particle's acceleration in a straight line is $a = (5\ m/s^3)t$. At t = 2 seconds its velocity is +17 m/s. What is its velocity at t = 4 seconds?

47.0 m $\int 5t\,dt = \frac{5}{2}t^2 + C$

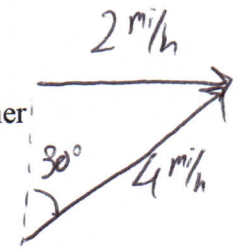
$\left(\frac{5}{2}\right)(4) + C = 17$

$C = 7$

$\left(\frac{5}{2}\right)(4^2) + 7 = 47$

plug in 4

4. A woman can row a boat at 4.0 mi/h in still water.



   a. If she is crossing a river where the current is 2.0 mi/h, in what direction must her boat be headed if she wants to reach a point directly opposite her starting point?

   30 degrees

   b. If the river is 4.8 mi wide, how long will it take her to cross the river?

   83.1 min $4.8/(4\cos 30) = 1.385h \times 60 = 83.1$

   c. Suppose that instead of crossing the river she rows 2.1 mi *down* the river and then back to her starting point. How long will she take?

   84 min $\frac{2.1}{4-2} + \frac{2.1}{4+2} = 1.4 \times 60 = 84 \text{ min}$

   d. How long will she take to row 2.1 mi *up* the river and then back to her starting point?

   min Same time

   e. In what direction should she head the boat if she wants to cross in the shortest possible time, and what is that time?

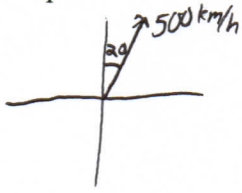
   90 degrees - straight across

   72 min

   $4.8/4 = 1.2 \times 60 = 72 \text{ min}$

5. A light plane attains an airspeed of 500 km/h. The pilot sets out for a destination 765 km to the north but discovers that the plane must be headed 20.0° east of north to fly there directly. The plane arrives in 2.00 h. What was the wind velocity vector?

   a. 192 km/h



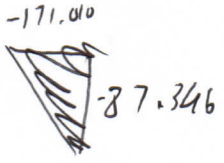
$\frac{765}{2h} = 500\cos 20 + x$

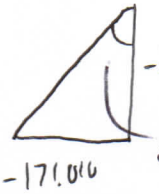
$x = -87.346 = y$ component of wind

$500 \sin 20 = 171.010 = x$ component

   b. -153 ° (where counterclockwise from the east direction is positive)

$\sqrt{(87.346)^2 + (171.0)^2} = 192$





$62.94_3 + 90 = 153°$