Unit 1: Equilibrium and Center of Mass

<u>FORCES</u>

What is a force? Forces are a result of the interaction between two objects. They push things, pull things, keep things together, pull things apart. It's really that simple, but this simple concept can be applied in many areas. It can range from the simple force of the floor holding you up right now, to the strong nuclear force that holds protons together in the nuclei of atoms.

Forces are vector quantities. Vectors have *direction* and *magnitude*. Therefore, saying "a force of 10 N" would not be sufficient information, nor a sufficient answer. You must also include that the force is directed upward, for example. We draw vectors as arrows in our free-body diagrams.

Forces are measured in Newtons. Remember, you always need to include units on your AP Mechanics test. The units for force are always Newtons, which can be more fundamentally expressed as $kg * m/s^2$.

Forces are always *on* something, *by* something. As discussed earlier, forces are the result of interaction between objects. The thing exerting the force is known as the **agent,** and the thing receiving the force is known as the **object.** For example, if I stand on the floor, the floor exerts a force on me. I am the object, the floor is the agent.

On the AP Mechanics exam, we're concerned with these primary forces:

<u>Gravitational Force</u>

Force that masses exert on each other. We'll delve further into this when we discuss Universal Gravitation, but usually in Mechanics gravitational force is just the force that the Earth exerts on objects. **This force always points down towards the Earth.** The equation used is:

$F_G = mg$, *where g is* $9.8\ N/kg$, *the specific constant with which Earth pulls on objects.*

Note that gravitational force is **also known as weight, and can be expressed as W.** Why is this important? Students will often confuse "weight" with "mass". Note that mass is independent of the Earth's gravitational field: your *mass* wouldn't change if you went to the Moon, but your *weight* would, since the Moon pulls on us with a different force and would have a different "g" constant as a result. **Always remember to have your mass in kilograms, because that is the SI unit for mass, and it is what the constant is made to be used with.**

Also, students confuse weight with mass when solving problems as well. Say we have the force of gravity set equal to another force (normal force, for example). **If the problem tells you the weight**

is 100 N, you don't need to use the above equation. On the other hand, **if the problem tells you an object has a mass of 10 kg, you would need to use the equation to find the force itself.**

Normal Force

The "normal" force is a force that a surface exerts on any object that is in contact with it. It is usually expressed as $F_N\ or\ N.$ It is **always** perpendicular to the surface applying the force (hence the term normal, which means orthogonal or perpendicular when discussing vectors) and is **only a pushing force.**

Tension Force

Somewhat self explanatory. Tension force is the force produced by the tension in a rope, chain, string, or something of the sort. It can be expressed as $F_T\ or\ T.$ **Note that tension force is only a pulling force.** Also, for *MOST* problems that show up on the AP Mechanics exam, ropes and strings are **assumed to be massless.**

Friction

There are two main varieties of friction, kinetic and static. Static friction **acts on objects that aren't moving.** If a box is on an incline, but isn't sliding down, it is being held in place partly by static friction. Kinetic friction **acts on objects that are moving.** It works to slow down objects **opposite** to the direction they are moving.

All frictional forces act parallel to the surface that applies them, which means that they are also always perpendicular to the normal force that surface provides.

The equation we use for frictional force is as follows:

$$F_f = \mu N,\ \textit{where } \mu \textit{ is the coefficient of friction between two surfaces, which is always less than } 1.$$

Since the coefficient is **almost always less than 1,** we know that it is harder to lift an object directly than it is to push it across a surface.

Applied

Any miscellaneous force applied by a person, dog, or any external force, usually just referred to as F.

EQUILIBRIUM

Equilibrium is when the sum of all forces on an object is 0 (they are "balanced"). This means that all the forces on the box are of equal magnitude and opposite directions, resulting in them cancelling and creating equilibrium.

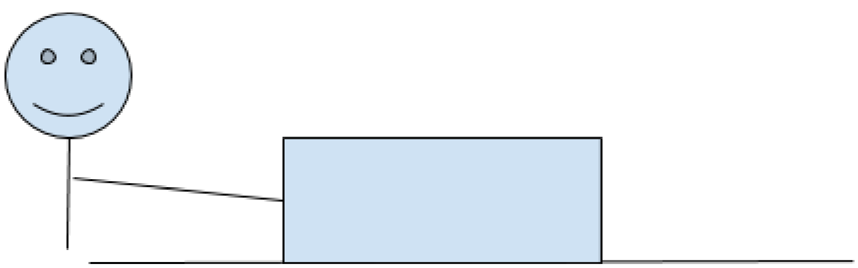
Newton's first law dictates that an object in motion will stay in motion, and an object at rest will stay at rest, unless an outside **unbalanced** force acts upon it.

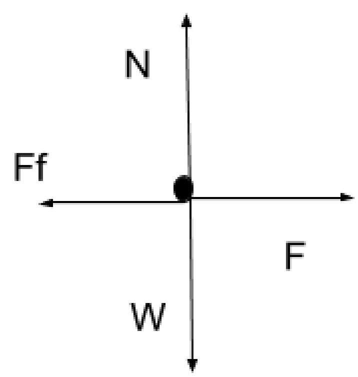
In other words, unbalanced forces **speed up** and **slow down** objects.

Therefore, an object with balanced forces can be either at rest **(static equilibrium)** or in constant motion **(dynamic equilibrium).** Since there are no unbalanced forces, there is no reason these objects will change how they are behaving.

VECTORS AND EQUATIONS FOR EQUILIBRIUM PROBLEMS

A 5 kg block sits on a table, and is being pushed right by a person at a constant speed on a surface with friction. Draw a free-body diagram and write force equations for the forces on the box. Then, solve for the normal force on the block.





A free-body-diagram (FBD) represents the force vectors on an object. The dot represents the objects, with arrows representing the force vectors. Since we know the object is moving at a **constant speed**, the forces must be balanced. We then divide up the two dimensions of vectors for our force equations. **Note that directions are independent.** An object can have unbalanced forces in the x direction, meaning an acceleration, and still have equal forces in the y direction. Since the normal force and weight are opposite, our y equation must be:

$N = W$

The forces must also be equal in the x direction, so our x equation is:

$F_f = F$

The problem asks for the normal force, so we apply the following equations to get our answer:

$N = W \text{ and } W = F_G = mg$

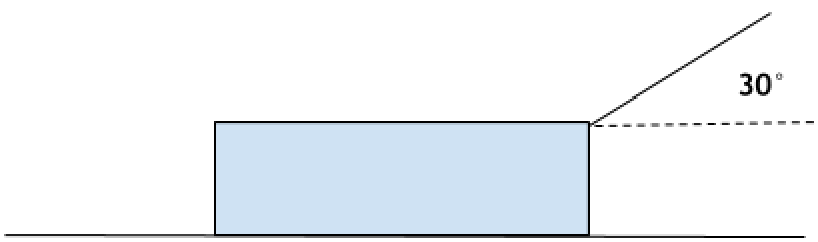
$N = mg \ = (5\ kg)(9.8\ N/kg) \ = 49\ N$

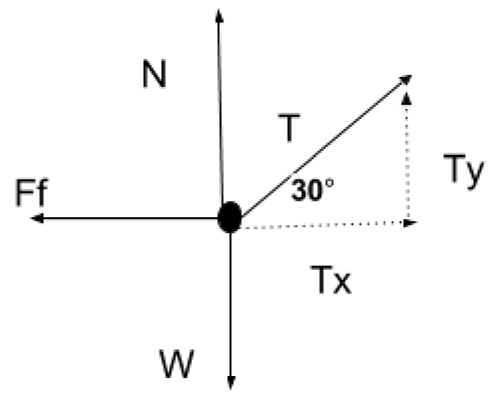
There are additional strategies to solving problems, especially in more complicated scenarios.

COMPONENT VECTORS

Take this problem, for example:

A block sits on a surface with friction, and is being pulled right by a rope at an angle of 30 degrees with the horizontal. Draw the free body diagram and write the force equations for this scenario.



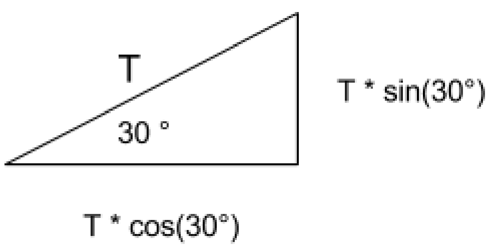


Notice that the tension force, T is at an angle. Since we are given the angle, we divide the force up into its **component vectors**, and include each component vector in either the y or x direction. So our force equations will be:

$Y\ Equation :\ N\ +\ T_y\ =\ W$

$X\ Equation : F_f = T_x$

But how do we know what $T_x\ and\ T_y$ are? We use geometry to divide up the components.

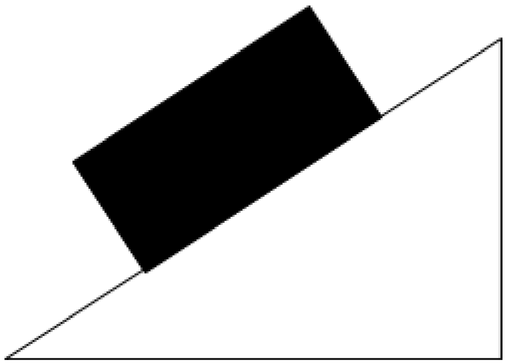


cos(30˚)=adjacent/hypotenuse, and sin(30˚)=opposite/hypotenuse. We rearrange this to solve for the components given in the triangle above. This way, we know that $T_x = T cos(30°)\ and\ T_y = T sin(30°)$. It is important that you draw a triangle every time you are given an angled force so that you can divide up the components correctly. Note that if you have solved for components and aren't given the angle, you can use $tan^{-1}(T_y/T_x)$ to solve for it (for this particular angle - if solving for the other angle use different trig function). Also, if you have solved for components and you need to solve for the tension force itself, you can use The Pythagorean Theorem: $T_x^2 + T_y^2 = T^2$.

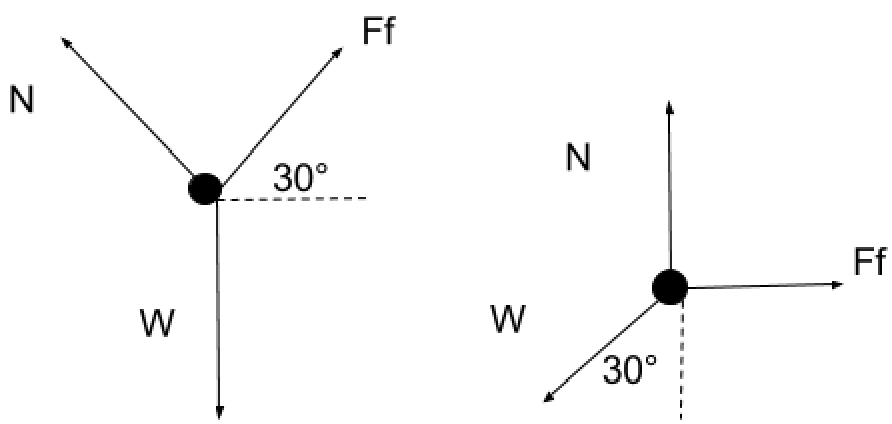
ROTATED COORDINATE SYSTEMS

In reality, we care about which forces oppose each other, so rotating free body diagrams is perfectly okay. What does rotating mean? Look at the following problem.

A box sits on an inclined plane (with friction), with a ramp angle of 30°. Draw the free body diagram and write force equations for the system.



Based on what you've learned thus far, you would draw a free body diagram in figure 1. But notice that there are two angled forces in this problem. **Since we know that F_f and N are perpendicular, we can rotate the free body diagram so that only one force is angled, like in figure 2.**



Note that this doesn't mean gravity is now at an angle, it simply means we have simplified the solving greatly by leaving only one angled force. Now the force equations are as follows:

$Y\ Equation : N = W_y = W cos(30°)$

$X\ Equation : F_f = W_x = W sin(30°)$

TORQUE & TORQUE EQUILIBRIUM

What is torque? Torque is the rotational analogue of force, simply a force exerted on an object in an attempt to rotate it about its axis.

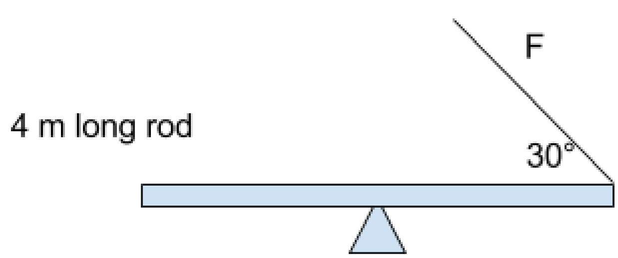
To understand torque, simply imagine a see-saw. Imagine that you and your friend, who weigh the exact same, are each on opposite sides of the see saw. Let's say that he sits a distance of 1 meter from the rotating point of the see-saw, and you sit a distance of 2 meters away, at the end of the see-saw. Which direction does it tip? If you've been on a see-saw before, or if you tried this experiment right now, **it would tip in your direction, since you are sitting further away from the center.**

Now let's imagine a second scenario. If you weigh twice as much as your friend, and you both sit 2 meters away from the center at both ends of the see-saw, what direction does it tip? **Again, it would tip in your direction, because you are applying twice the force**.

What we have learned from this hypothetical scenario? **That torque is a product of both force and distance from the axis of rotation.**

This is denoted on your formula sheet as a cross product: $\tau = r \times F$. This is equivalent to $\tau = rF\sin\theta$, *where* θ *is the angle between the force and the radius, and r is the distance to the axis of rotation.* Torque is measured in Newton-meters, and is donated with the variable Tau. **Always make sure that your radius is in meters.**

Why is the angle important? Because we only care about the component of the force that is perpendicular to the radius. For example, what is the torque applied by force F in the system below? We only care about the y-component of the force, so we first do Fsin(30°), and then multiply by the distance to the axis of rotation, which is 2 meters (half the length of the 4m long rod). So the torque applied by F in this problem is simply 2Fsin(30°), which just ends up being F.



For torque problems, we draw the vector directly where it acts on the system. Instead of x and y being balanced like in force equilibrium, we balance clockwise and counterclockwise torques in torque equilibrium. **Counterclockwise torques are positive, clockwise torques are negative.**

CENTER OF MASS

Center of mass is a very intuitive concept. Let's say you have a meter stick with a constant mass distribution. Where would you put your finger so that the meter stick doesn't fall when you balance it there? You would put it right in the middle.

Now let's say you have a hammer, obviously with a weighted head. Where would you put your finger so the hammer doesn't fall when you balance it? You would put it much closer to the heavier weighted head.

The easiest way to learn center of mass is to practice with the formula. The formula on the sheet is as follows:

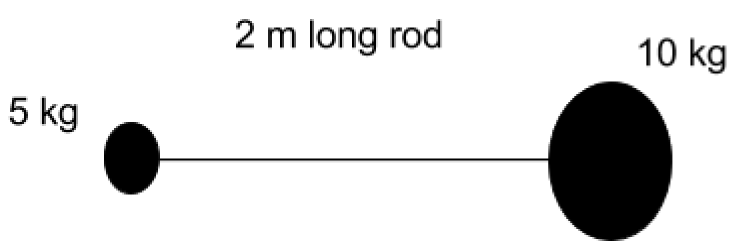
$$r_{cm} = \frac{\Sigma mr}{\Sigma m}$$

It can also be expressed as:

$$X = \frac{m_1 x_1 + m_2 x_2 + \ldots\ldots + m_n x_n}{m_1 + m_2 + \ldots\ldots\ldots + m_n} = \frac{\Sigma m_i x_i}{\Sigma m_i}$$

With each discrete mass in the system. Take a look at the following problem:

Find the center of mass of this system.

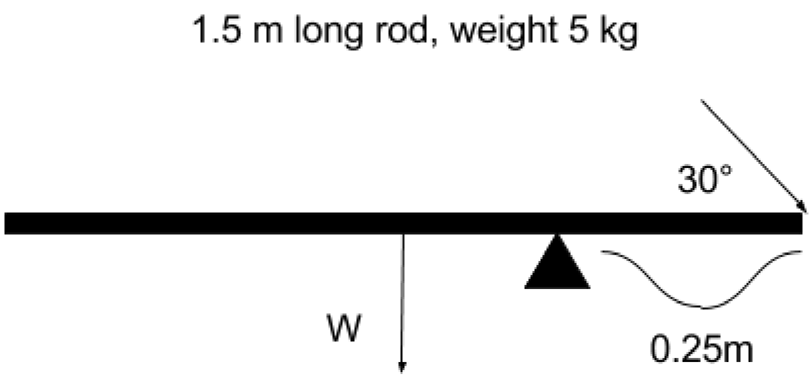


Note that we can measure that radius used in the formula from <u>anywhere</u> <u>as</u> <u>a</u> <u>reference</u> <u>point</u>, as long as we keep it consistent. The formula will then return the distance of the center of mass from that point. For example, if we use the left edge as a reference point, we have
R = (5kg)(0m) + (10kg)(2m) / (5kg + 10kg) = **1.333 meters from left edge.**
If we use the right edge as a reference point, we have
R = (5kg)(2m) + (10kg)(0m) / (5kg + 10kg) = **0.666 meters from right edge.**
Either reference point returns the right answer.

ONE LAST TORQUE CONCEPT

Now that you know exactly what a center of mass is, it's important to know one last torque concept. **The force of gravity of an object always acts on its center of mass.** Why is this important? Take a look at this problem.

What force must be applied at the right end of the rod in order to have this system at equilibrium?



Since we know that the weight is at the center of mass, it actually applies a counterclockwise torque on the system. Since the bar is 1.5 m long, the center of mass is **0.75 m from the right edge,** making it **0.5 m from the axis of rotation.** Applying our equations leads us to the answer:

$\tau_W = \tau_F$

$rW\sin\theta = rF\sin\theta$

$(0.5\ m)(5\ kg)(9.8\ N/kg)(\sin 90°) = (0.25\ m)(F)(\sin 30°)$

Solving for F yields **F = 196N**