Unit 2: Kinematics

There are very few sample problems in the lecture: Doing the worksheets and reading the solutions will help you actually understand how to solve the problems - refer to this for review and to understand the content.

ONE DIMENSIONAL KINEMATICS

Kinematics is the study of motion, without regard to the forces that caused the motion.

When we study one dimensional kinematics, we are concerned only about one frame in which the objects are moving, for example a car accelerating forward, a kid riding a bike at a constant speed, etc.

It is important to first define several key quantities:

Position (x or s): Where you are, with respect to the origin at a given time. For example, if my house is defined as the origin (and to the right of my house is positive and to the left is negative) then I can say that I am at 2 meters, which is 2 meters to the right of my house.

Distance: How far you have traveled in total. Distance has no direction, and thus **cannot be negative**. If I walk to school (4 miles away) and then walk back home, **the distance I have traveled is 8 meters.**

Displacement (Δx): Your change in position during a given time frame, simply defined as $x - x_0$. If I walk to school and then walk back home, **my displacement is 0 meters.** At the beginning I am at position 0, and at the end I am at position 0.

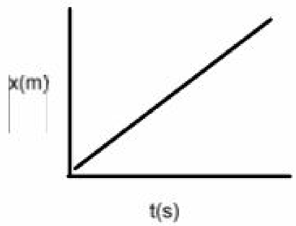
Velocity: How fast you are traveling, and which way. In one dimension, velocity can be either positive or negative, indicating that you are either moving forward or backward. It is expressed as $\Delta x / \Delta t$, and is measured in meters per second (m/s). If you have a positive displacement in a given time interval, you have a positive velocity. If you have a negative displacement in a given time interval, you have a negative velocity, since time is always positive. **Velocity is also the rate of change of position,** meaning that if you have a function that expresses position with respect to time (x(t)), then taking the derivative of that function will give you v(t).

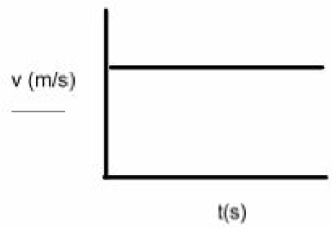
Speed: How fast you are traveling, and can be found with the absolute value of velocity.

Acceleration: The rate of change of velocity (meaning you can take the derivative of velocity to get acceleration). When you have a constant velocity, it is not changing, which means you have no acceleration. If the velocity is increasing, you have a positive acceleration. Note that an object **can have a positive velocity, but a negative acceleration.** This means an object is moving in the positive direction, but is slowing down while doing so.

Graphical Relationships

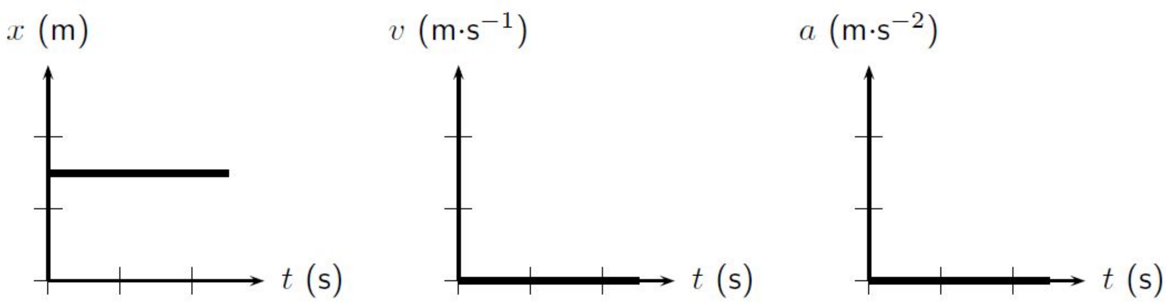
Variable (vs time)	Slope	Y-intercept	Area under curve
Position	Velocity	Starting Position	-
Velocity	Acceleration	Initial Velocity	Displacement
Acceleration	Jerk	Initial Acceleration	Change in Velocity



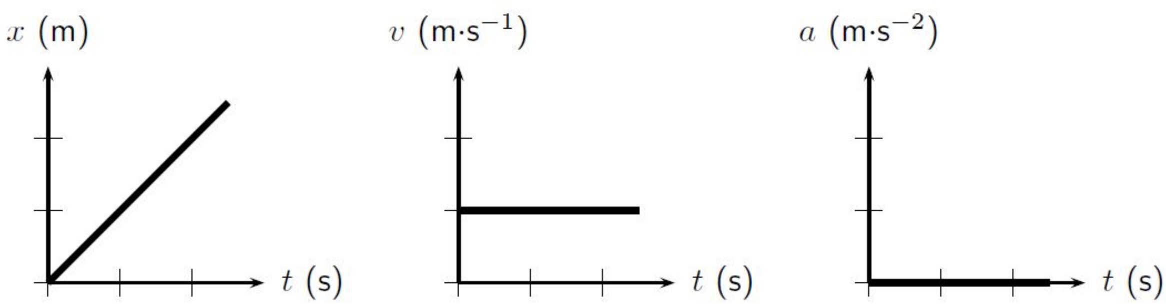


UNDERSTANDING GRAPHS OF UNIFORM ACCELERATION AND CONSTANT VELOCITY

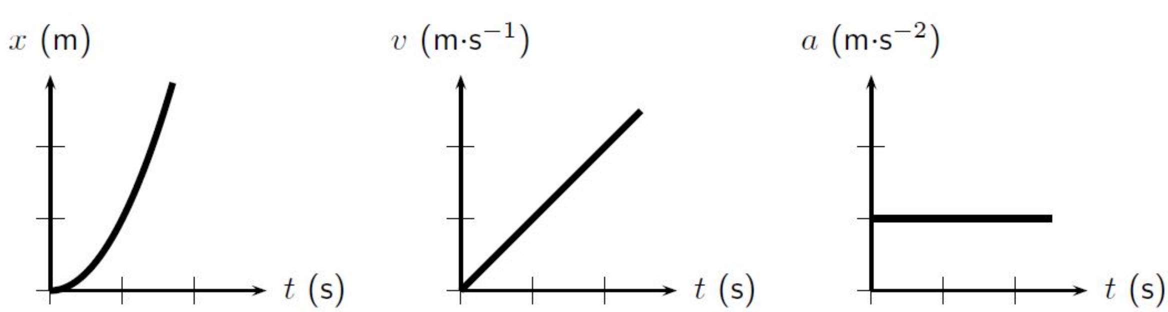
Stationary Object



Uniform Motion



Motion with constant acceleration



In the first line of graphs, the object is sitting still at some given position. Therefore its velocity is not changing, and neither is its acceleration.

In the second line of graphs, we have a constant positive velocity. The rate of change of position is velocity, and since it is a constant velocity, we have 0 acceleration and a constant slope in the position graph.
When the velocity is constant, we can use the equation $x = vt + x_o$. This is simply the equation of a line, applied to the position graph.

In the third line of graphs, we have a uniform positive acceleration. This means the velocity is increasing linearly with a slope of a, meaning we can apply the equation $v = at + v_0$. The position function is parabolic, and modeled with the equation $\Delta x = \frac{1}{2}at^2 + v_0 t$. You'll notice that if you take the derivative of the previous position equation, you get the velocity equation, and if you take the derivative of the velocity equation, you get the acceleration equation.

The "Famous Five" Equations

$x = vt + x_o$

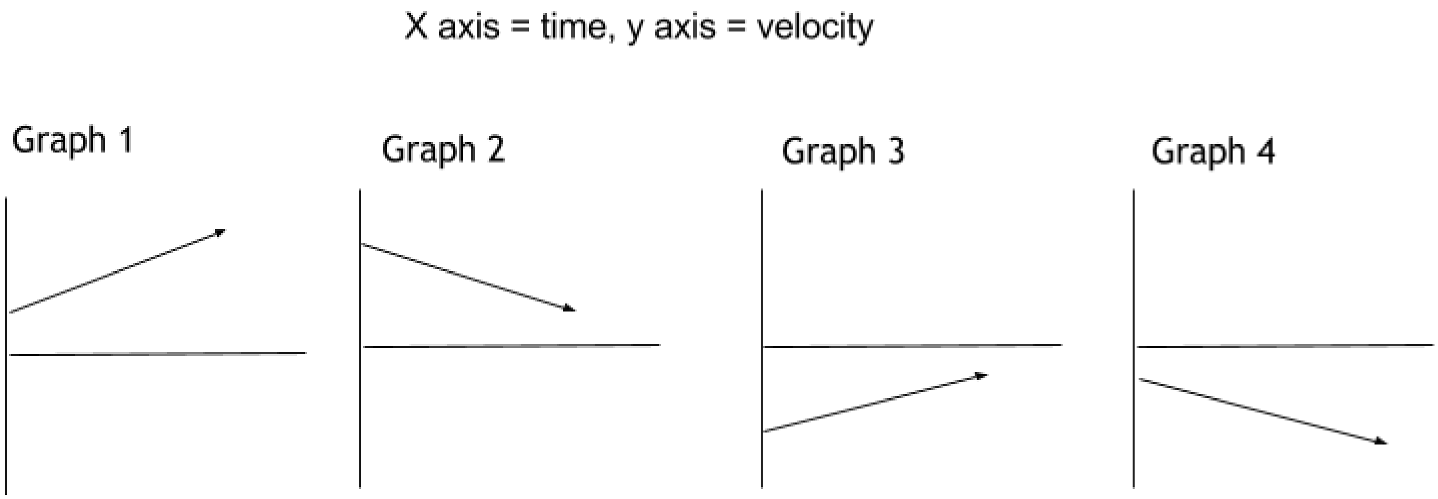
$$\boxed{\begin{gathered} x = 1/2at^2 + v_o t + x_o \\ v = at + v_o \\ 2a(x - x_o) + v_o^2 = v^2 \end{gathered}}$$

$x = 1/2t(v + v_o)$

These are the equations we have so far, with the last two on the bottom right being derived by combinations of the rest. Note that if you don't have acceleration you can use the last one, and if you don't have time given you can use the second to last one.

Remember: The equation on the left is ONLY for constant velocity, and the equations on the right are ONLY for constant acceleration. If the acceleration is changing, you can use calculus (remembering that derivative of position is velocity, integral of acceleration is velocity, etcetera) to solve the problems.

MORE GRAPHICAL ANALYSIS



Graph 1
Velocity is increasing, which means acceleration is positive.
The object is speeding up, because the graph is moving away from the origin.
Graph 2
Velocity is decreasing, which means acceleration is negative.
The object is slowing down, because the graph is moving toward the origin.
Graph 3
Velocity is **increasing,** which means acceleration is positive.
But, the object is **slowing down,** because the graph is moving toward the origin.
Graph 4
Velocity is **decreasing,** which means acceleration is negative.
But, the object is **speeding up,** because the graph is moving away from the origin.

Those last two graphs can be tricky in terms of the vocabulary (speed vs. velocity) and the AP exam will try and trick you with these quite often.

AVERAGE SPEED V. AVERAGE VELOCITY

Average velocity is the total displacement divided by the total time.
Average speed is the total distance divided by the total time.

Take a look at this problem, for example:
Carly drives her Prius 10 miles to work at a constant speed of 10 mph, but then realizes she forgot her favorite physics textbook at home, so she drives back, but there's no traffic so she rocks the speed up to 20 mph. Then she is too lazy to drive back, so she ends her trip at home. What is Carly's average velocity and what is her average speed for the duration of this round-trip?

Solution: Average Velocity

A lot of students would want to answer 15 mph, because that is the average of 10 mph and 20 mph. But, remember average velocity is total displacement over total time. Since her **displacement is actually 0 (she started and ended at her home),** her average velocity is simply **0 mph.**

Solution: Average Speed

Again, a lot of students **would want to answer 15 mph,** being the regular average. But, we cannot directly average these numbers! Think about it: she is actually traveling at 10 mph **twice as long** as she is at 20 mph, since she'll take twice as long to go the same distance of 10 miles. So we use the previous formula - total distance over total time. The **total distance is 20 miles, since she went 10 miles one way and 10 miles back.** The total time is a bit different. On the way there, she took 1 hour to complete the trip (10 miles per hour, going 10 miles). On the way back, she took half an hour to complete the trip (she was going double the speed). Therefore, the answer is **20 miles / 1.5 hours, or 13.333 miles per hour.**

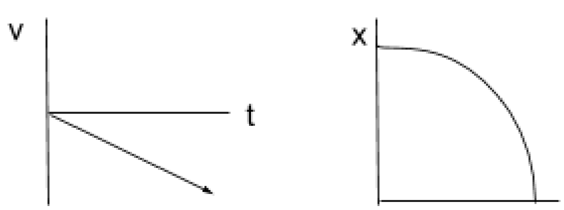
VERTICAL TRAVEL

Things that go up, come back down. Objects don't fall at a constant speed - right when you drop an object, it's not moving at all, but by the time it's about to hit the floor it is moving fast. **This means that falling objects accelerate naturally.** The Earth's gravitational field creates a constant acceleration of **9.8 m/s^2.** For problem solving purposes, a velocity **upward (away from the Earth)** is positive, while a velocity **downward (towards the Earth)** is negative.

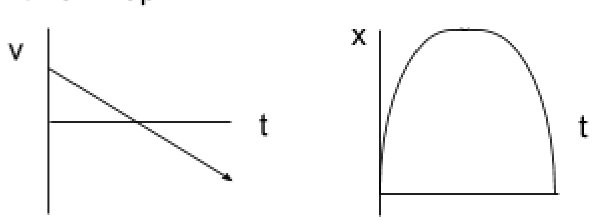
KEY POINTS:

When an object is at the top of its path, it is at rest - a velocity of 0, but still a constant negative acceleration of 9.8 m/s^2. We can use the same "famous five" equations with vertical travel - and use -9.8 as the "a" value.

Ball dropped from a height



Ball thrown up



Slope of v/t
graph = -9.8
V = 0 at highest
point

PROJECTILE MOTION: TWO DIMENSIONAL KINEMATICS

What is a projectile? A projectile is an object that has only the force of gravity on it throughout its time of flight. A common misconception is that an object moving upward has an upward force on it - when in fact this isn't the case. An object thrown upward will accelerate downward, meaning it is slowing down until it reaches its apex, and then starts to speed up downward.

The only difference between a projectile and the vertical travel addressed earlier is that **a projectile moves in two dimensions.** This means that - while it is accelerating downward in the y-axis, it is also moving in the x-direction. But, since there is only a force in the y direction, **the x velocity of the projectile doesn't change - the initial x velocity given to the projectile is maintained throughout flight.** This is rather counterintuitive to most people.

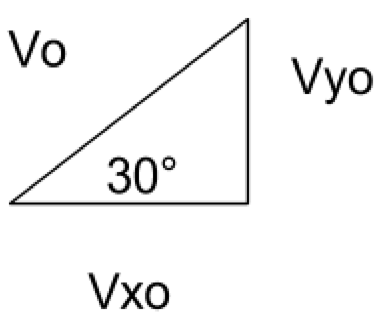
To understand this, imagine the following scenario. A bullet is dropped from a height of 1 meter. At the exact same time, a bullet is fired out of a gun, and the barrel of that gun is held at 1 meter. Which bullet hits the ground first?

Answer: They hit the ground at the exact same time. The presence or absence or variation in x velocity has no effect on the time in the y direction. The only thing that affects the y direction are the things in the y direction: initial y velocity, acceleration in the y, and height in the y. Those 3 things are the same in both scenarios, so it makes sense that they hit the ground at the same time.

We use the simple famous five equations to govern the laws of projectile motion.
$x = V_{xo}t$ and $y = 1/2gt^2 + V_{yo}t$ *with g being* $-9.8\ m/s^2$, *the acceleration due to gravity.*

Problems may ask you to solve for the maximum height of the projectile, for which you only need the y direction. Also, problems may ask for the "range" of a projectile. That's simply how far it travels in the x before hitting the floor. We can solve the time it takes in the y equation, and plug that same time into the x equation (that is the time that the projectile is moving forward).

Often, a velocity is given with an angle. We divide this up the same way we divided forces up, and plug each velocity into our respective equations, as follows. The example is for an object launched with a velocity of Vo, with an angle of 30 degrees to the horizontal.



RELATIVE VELOCITY

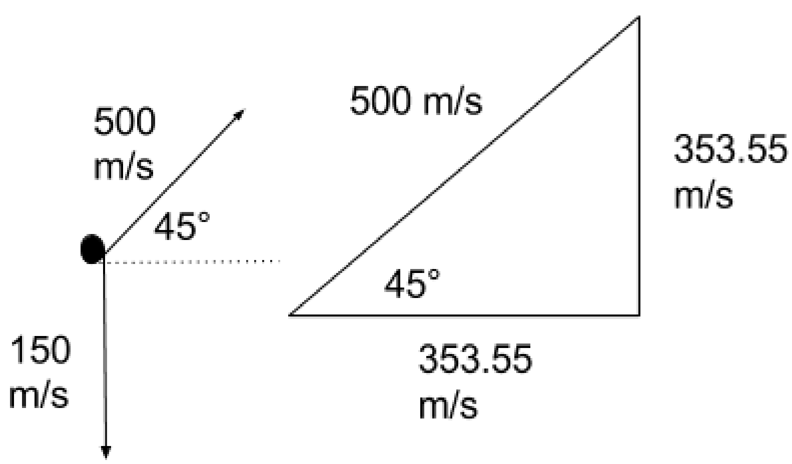
How do we define velocity? Usually we define velocity relative to the Earth, which is not mentioned. When a car is moving at 5 m/s, it is moving at 5 m/s **relative to the Earth.** But, velocity can be defined relative to other things as well. For example, if I'm on a train and walking forward at 5 m/s, I'm walking at 5 m/s relative to the train. My speed relative to the ground is much greater, in fact is the addition of the velocity of the train and my velocity.

This applies in all relative velocity scenarios. For example, if a car next to me is moving at 45 m/s, and I'm moving at 42 m/s, I'm actually moving at -3 m/s relative to that other car.

The AP test will have more complicated problems than this, although they don't come up that frequently.

Take the following problem for example:

An airplane is flying northeast at an airspeed (relative to the air) 500 m/s, but there is a 150 m/s wind going south. What is the airplane's actual speed relative to the ground?



First, we divide up the 500 m/s into its constituent North and East velocities (using trig; it's done the same way the projectiles are, see previous page): 353.55 m/s North, 353.55 m/s East.

Then, since the 150 m/s wind is opposing the North direction, we subtract that from the North velocity, getting a new North velocity of 203.55 m/s, and an East that is still 353.55 m/s. Then we input these back into the triangle and use Pythagorean's Theorem: $v^2 = 353.55^2 + 203.55^2$ and get a velocity of 407.96 m/s. We also need the angle, but since it's a right triangle we can simply do tangent inverse of the North velocity over the East velocity (tangent is opposite over adjacent), and get an angle of $tan^{-1}(203.55/353.55) = 29.93°$.