Unit 3: Dynamics and Gravitation

DYNAMICS

Dynamics combine the concept of forces with our understanding of motion (kinematics) to relate forces to acceleration in objects.

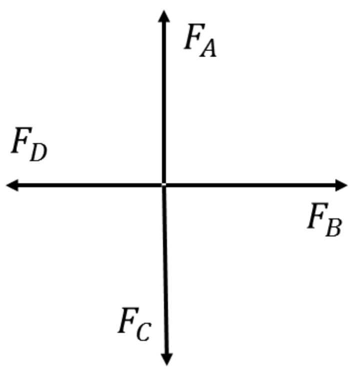
Newton's Second Law states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. More simply, it is often denoted as F=ma.

The formula on the sheet is written as $\boldsymbol{F}_{net} = \sum \boldsymbol{F} = ma$.

To solve problems with Newton's second law, we simply extend the application of free body diagrams from Unit 1. Whereas then we were dealing only with balanced forces, we now also will have unbalanced forces. **Unbalanced forces can only speed up or slow down an object, thus they create an acceleration**, **balanced forces maintain constant speed or keep an object at rest.**

To demonstrate how we can easily apply this, imagine the following free body diagram, with force values as follows:

$F_A = 10N, F_B = 15N, F_C = 10N, F_D = 10N$



In the y direction, the forces are **balanced**, which means our force equation is simply $F_A = F_C$.

In the x direction, the forces are **unbalanced**, which means our force equation is $F_B = F_D$.

If the forces are unbalanced, then we write a **net force equation,** which in this case is

$F_{net} = F_B - F_D = 15N - 10N = 5N.$

We have determined that there is a net force of 5N right (in the positive x direction).

To write a net force equation, determine which side is greater (left or right side), sum up all the forces on that side, and subtract the forces that oppose it. It is easiest to do this instead of getting mixed up with negatives and positives. **When stating the net force, we state the magnitude and the direction (force is a vector),e.g. 5N right, 10N left.**

How can we apply this to Newton's second law? Well, if these forces were acting on a 5 kg block, then we now solve for its acceleration: $a = \frac{F}{m} = \frac{5N}{5kg} = \frac{1m}{s^2}$.

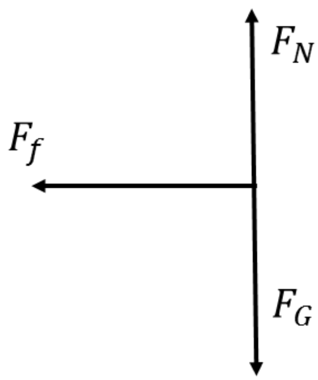
We now know that the block is accelerating 1 m/s^2 right.

Here's an example of a harder problem. Don't worry if you're not able to figure it out immediately, but you should follow and understand the solution.

A 5 kg block is moving right along a frictionless surface at 5 m/s, until it suddenly encounters a surface with a coefficient of friction of 0.10. How far does the block travel before stopping on the surface with friction?

Solution:

First, we should draw our free-body diagram. If you recall from Unit 1, friction **always opposes the direction of motion,** so it will be pointed right.



We know that the y direction forces are balanced, resulting in a force equation of $F_N = F_G$.

We know that the x direction has **a net force left** ($F_f > 0$), with a net force equation of $F_{net} = F_f$.

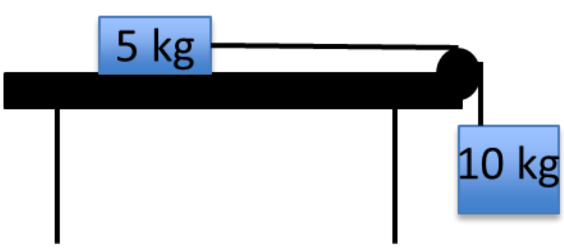
We now apply the equations that we have learned to get $ma = \mu F_N$, and we know that $F_N = F_G = mg$, so we get $ma = \mu mg\ and\ a = \mu g = 0.98\frac{m}{s^2} to\ the\ left.$

We know the block ends at a velocity of 0, starts at a velocity of 5 m/s, and has an acceleration left of 0.98 m/s^2. Applying the equation $v_f^2 = 2a\Delta x + v_0^2$, we get that $\Delta x = -\frac{v_0^2}{2a} = -\frac{\left(5\frac{m}{s}\right)^2}{2\left(-0.98\frac{m}{s^2}\right)} = 12.76\ m$. **Remember to plug in a negative acceleration,** because we got a positive acceleration going **left**, which in kinematics must be negative because right is positive and left is negative.

That's it for the concepts of force diagrams, but there are plenty of tricky problems with common misconceptions that we can show through problem and solution. Let's start with a pair of problems.

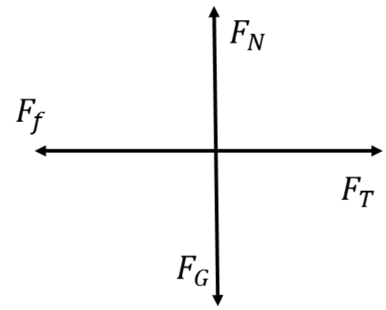
Problem A: A 5 kg block is sitting on a table with a coefficient of friction of 0.2. There is a rope pulling the block right with a force of 98 N. What is the acceleration of the block?

Problem B: A 5 kg block is sitting on a table with a coefficient of friction of 0.2. There is a rope that runs over a frictionless pulley that is attached to a 10 kg block (see diagram).



Solution:

a) First, we draw our free body diagram for the block in the picture:



Next, writing our force equations gives us $F_T > F_f$ in the x and $F_N = F_G$ in the y, with a net force equation of $F_{net} = F_T - F_f$.

Plugging in our equations and values, we get:

$$ma = F_T - \mu F_N$$

$$ma = F_T - \mu mg$$

$$(5\ kg)a = 98\ N - (0.2)(5\ kg)(9.8\frac{N}{kg})$$

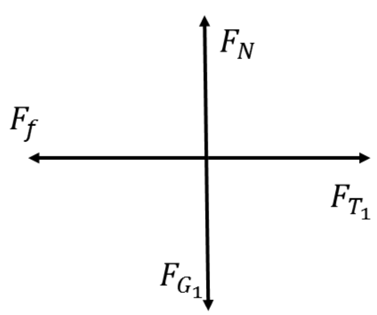
$$a = 17.64\frac{m}{s^2}\ right$$

b) The most common misconception with problem B is that, since the bottom box weighs 98 N, it is applying that force on the upper block (just like in part A the box had 98 N applied on it) and the acceleration will be the same: 17.64 m/s^2.

But this actually is not the case, because the box on the table has **much less than 98 N** being applied on it. Let's draw out the free-body diagrams for both boxes.

The top box will have the same free body diagram and equations as the top box in problem A, but we'll re-label the variables with 1s and 2s so that we don't confuse the forces for both boxes.

Top Box



Bottom Box

This results the equations being $F_{T_1} > F_f$ in the x and $F_N = F_{G_1}$ in the y, with a net force equation of $F_{net_1} = F_{T_1} - F_f$.

The free body diagram for the bottom box would just be the tension and gravity forces, resulting in the force equations as follows:

$F_{T_2} > F_{G_2}$, and a net force equation of $F_{net_2} = F_{T_2} - F_{G_2}$. You see that the tension is **less than** the gravitational force, meaning it is less than 98 N, so the answer cannot be the same as problem A.

After both of these equations are implemented, we have $(5\ kg)(a_1) = F_{T_1} - \mu(5\ kg)(g)$ and $(10\ kg)(a_2) = F_{T_2} - (10\ kg)(g)$.

Now we have two equations that each have their own individual variables. But there are a few conceptual assumptions that we can make to simplify the problem. Firstly, if the boxes are tied together, one box cannot accelerate faster than the other. This means that $a_1 = a_2$, which eliminates one extra variable.

The second is an assumption often made in classical mechanics, which is that the tension within a rope is the same throughout its entire length. This means that the tension pulling right on the top block is the same tension that is resisting change in motion in the bottom block, meaning that $F_{T_1} = F_{T_2}$. Now, we have two equations and two variables (the same equations from before, except there aren't two different as and two different Fts, they're just Ft and a in both equations. Applying our algebra, plugging in the values of μ and g, we solve a system of two equations and get $a = 5.88\frac{m}{s^2}$.

ALTERNATIVE SOLUTION AND CONCEPT

In the long solution for problem B, we had to set up two different force diagrams and have two different F_{net} equations, but what if we didn't have to do that? What if the tension force wasn't even a factor? We had two different **objects** in the first solution, and we were only concerned with the forces on each object.

Consider this: what if the entire system of both blocks and the rope connecting them was our object! Then there would be no tension forces on the "object" thereby eliminating the tension completely from the problem (we can only do this because the tension is the same throughout the rope).

Drawing free body diagrams for such a scenario is fruitless, because directions are difficult to define. Instead let's just look at forces that oppose each other: gravity is trying to "rotate" the system right, and friction is resisting the pull that gravity has on the system. So let's write the equation for that:$F_{net} = F_G - F_f$.

The important part with this method is knowing which masses to plug in: Let's say the mass of the top block is m_1 and the mass of the bottom block is m_2. The equation would be as follows:

$$F_{net} = F_G - \mu F_N$$

$$(m_1 + m_2)(a) = (m_2)(g) - \mu(m_1)(g)$$

For the mass in F=ma, we use the total mass of the system, hence both masses. For the mass in gravity, only the bottom block has an unbalanced force. For the mass in friction, only the top block is in contact with the surface.

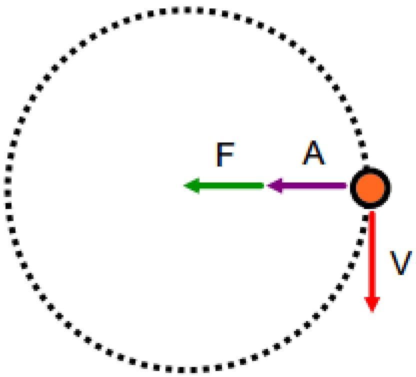
Plugging in the values gets us:

$$(15\,kg)(a) = (10\,kg)(9.8\frac{N}{kg}) - (0.2)(5\,kg)(9.8\frac{N}{kg})$$

$a = 5.88\frac{m}{s^2}$, which is the same value we got in the previous solution.

UNIFORM CIRCULAR MOTION

Uniform Circular Motion deals with objects that are revolving **not objects that are rotating.** Uniform circular motion is also known as centripetal motion. Centripetal means towards the center of a circle, which refers to the acceleration of the object.

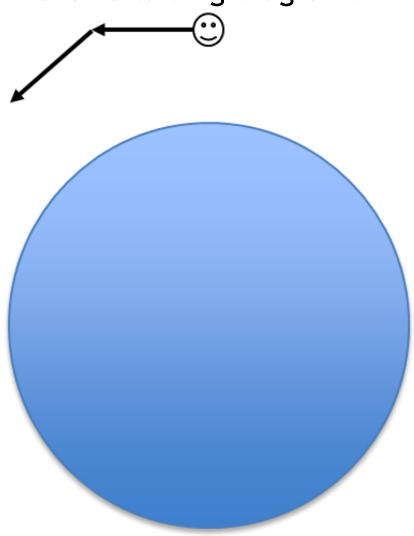


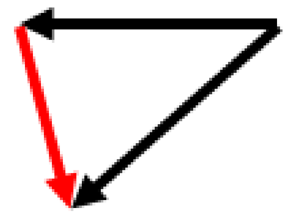
Key Points of Centripetal Motion

- Constant acceleration towards the center of the circle
- Force towards the center of the circle (force and acceleration go together)
- Velocity tangential to the circle (perpendicular to acceleration)
- Constant speed throughout motion
- **Period** is the time it takes to complete one cycle, variable is T.
- **Frequency** is the cycles completed in 1 second (taking the inverse of period gives you frequency and vice versa), the variable is f.

But why does the object move in a circle?

What you've learned in dynamics so far shows that if there is an unbalanced force, then the object will accelerate in that direction. This still holds true here: if we dropped an object in space near the Earth, it would simply accelerate towards the Earth. **But, if we give it a large enough tangential velocity**, the acceleration is **perpendicular** to the velocity vector, and therefore cannot change the speed of the object. It continues to pull the object inward and creates a circular path, as shown in the following diagram.





The object is **accelerating** because acceleration is defined as change in velocity – and the object's velocity is changing, but **its speed is not.** This is because the direction is constantly changing. As you can see in the diagram on the right, when the vectors are lined up, the red vector is change in velocity, or acceleration, and it points towards the center of the circle.

SOLVING CIRCULAR MOTION PROBLEMS

When solving circular motion problems, we once again will need to write "F_{net}" equations, but for centripetal motion, we'll write it as "F_c" instead – taking into account forces that point towards the center of the circle. Since F_c is just a supplement for F_{net}, it still equals ma.

But how do we calculate the centripetal acceleration of an object?

On your formula sheet you'll see that $a = \frac{v^2}{r}$, where v is speed. Which, by combining our formulas, shows that $F_c = \frac{mv^2}{r}$.

Before getting into the diagrams, let's do a quick practice problem manipulating the variables of centripetal motion:

A 5 kg Jigglypuff is moving in a circle with a radius of 10 m at 30 rpm. What is the centripetal force felt by the Pokemon?

Solution:

Our formula is $F_c = \frac{mv^2}{r}$.

We know the mass and the radius, but not the speed. However, we are given the rotations per minute. First, we can find the rotations per second, which gives us the **frequency.** That is 0.5 cycles per second. This means that the object takes 2 seconds per cycle (the **period**). How does this help us find the speed? Well, we know how long a cycle is in meters because we can find the circumference of a circle with 2πr and divide it by the time it takes to complete a cycle (speed = distance/time). We then proceed with the full calculation as follows:

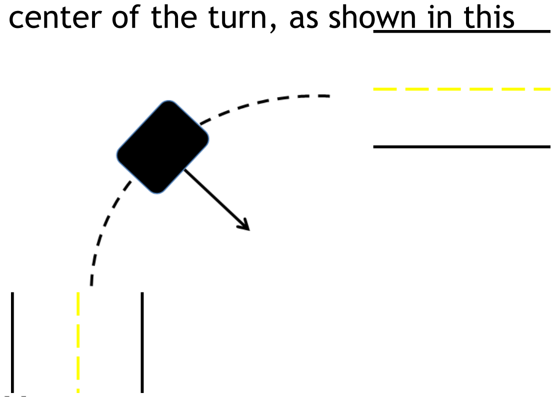
$$v = \frac{2\pi r}{T} = \frac{(2)(\pi)(10\ m)}{2\ \ seconds\ per\ cycle} = 10\pi\ m/s$$

$$F_c = \frac{mv^2}{r} = \frac{(5\ kg)\left(10\pi \frac{m}{s}\right)^2}{10m} = 493.48\ N$$

Let's try one more, which is a bit more of a real world problem.

Pikachu is driving the batmobile around a left turn with a radius of 7.5 meters, but he is driving on ice so the coefficient of friction between the road and the tires is 0.1. What is Pikachu's speed throughout the turn? From the driver's seat perspective, what direction is the centripetal force?

Solution: The tires are rolling, but there is friction that is keeping the tires from slipping in their horizontal axis. That frictional force points towards the center of the turn, as shown in this diagram.



Therefore, we know that $F_C = F_G$

Plugging in our formulas leads us to the answer:

$$\frac{mv^2}{r} = \mu F_N$$

$$\frac{\cancel{m}v^2}{r} = \mu \cancel{m} g$$

Note that the mass was not given, because in this particular problem we don't need to know the mass to solve the problem.

$$\frac{v^2}{7.5\ m} = (0.10)(9.8\frac{m}{s^2})$$

$$v = \sqrt{(7.5m)(0.10)(9.8\frac{m}{s^2})} = 2.711\ m/s$$

Looking at the diagram, we see that in a left turn, the centripetal force will be left from the perspective of the driver.

But when we take a sharp left turn, objects in our car go right, even though the centripetal force is LEFT! Why does this happen?

If you recall Newton's first law, objects resist changes in motion. Objects that are going straight want to keep going straight, and since the car takes a sharp turn, it seems like they are moving right, when in fact they are just trying to go straight while the car is turning left. If you don't understand what I mean, take a sharp turn with your car sometime and see what happens to objects in your car.

NON UNIFORM CIRCULAR MOTION

We don't need to worry too much about problems involving non uniform circular motion. The most common example that shows up with these types of problems are things revolving, but vertically.

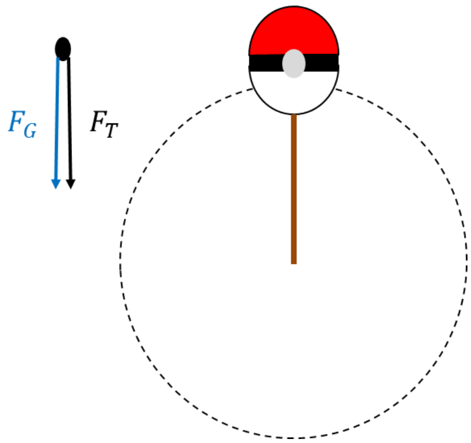
The speed of the object changes in non uniform motion, which means that the centripetal acceleration (and therefore the centripetal force) also changes. The same formula applies, but with a changing force. A classic example is spinning a ball on a string vertically – at the bottom, tension is towards the center of the circle, but gravity is away. At the top, tension and gravity are both down. The acceleration and force varies throughout the motion of the object.

Let's try a common problem that uses these concepts:

Ash is spinning a Pokeball on the end of a rope in a vertical circle of radius *r*. He wants to know the minimum possible velocity at the top of the circle so that the ball completes a full circle.

Solution:

For the minimum possible velocity, it is assumed that the tension force at the top is 0 (rope completely slack at the top, has tension everywhere else).



Therefore, at the top of the motion the equation is $F_C = F_T + F_G = \frac{mv^2}{r}$. Since Ft is 0 at the top of the motion, we just have $\cancel{m}g = \cancel{m}v^2/r$. Therefore, the minimum speed is $v = \sqrt{gr}$.

<u>UNIVERSAL GRAVITATION</u>

Newton's law of **universal gravitation** states that

"Two bodies in the universe attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them."

This can be represented with the formula:

$$F_G = \frac{Gm_1m_2}{r^2} \qquad G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{Kg^2}$$

So ALL OBJECTS, regardless of mass and when placed at a distance r, exert a gravitational force on each other. But, because of the very small G constant, the force exerted is miniscule between regular objects, and the main form of gravitation that we study is between planets, stars and other objects in space.

NOTE that since both the masses are in the formula, the force that one mass exerts on the other and the force that the second mass exerts on the first mass are the EXACT SAME. Newton's third law also supports this.

Also, when solving problems with Universal Gravitation, make sure the radius is from the center of mass of one object to the center of mass of the other. If the Earth is one of the objects, make sure you take the distance from the surface of the Earth to the object and add the radius of the Earth as well. The problems on the worksheet include more advanced solving which combine gravitation and circular motion.

Earth's mass (m_e) = 5.97×10^{24} kg

Earth's radius (r_e) = 6.37×10^{6} m