

Unit 4: Energy, Work, Momentum

ENERGY

Objects in mechanics have “mechanical energy” - which can be sorted into **kinetic** and **potential** energies.

- **Potential energy** is stored energy based on position - an object at a certain height has **gravitational potential energy** that can be converted into other forms of energy. It is denoted with the variable U and gravitational potential energy has the formula $U_G = mgh$, where m is the mass in kilograms, g is the constant 9.8 N/kg , and h is the height that the object is at in meters (with respect to the reference point).
- **Kinetic energy** is the energy objects have while in motion. All objects in motion have kinetic energy, and it has the formula $E_K = \frac{1}{2}mv^2$, where mass is again in kilograms and v is the speed of the object in m/s .
- **Kinetic** and **Potential** energy fall under “mechanical energy” whereas thermal energy is known as internal energy. Conservative forces conserve mechanical energy (e.g. gravity, energy is conserved when gravity acts on a falling block). Non-conservative forces don't conserve mechanical energy (friction, air resistance - energy is still conserved but becomes heat)

The law of conservation of energy states that the total energy of a system doesn't change unless an acted on by an external force.

For example, even when an object slows down by friction, it loses kinetic energy, but gains thermal energy, **and the total energy of the system remains the same.**

This concept is what we use to solve problems, like this next one:

Charizard is holding a Pokeball at a height h . He drops it: What is the Pokeball's speed right before it hits the floor in terms of the given variables and fundamental constants?

Using our previous knowledge, we could solve the problem with kinematics. But, as you'll see, energy is a much easier, more straightforward way to solve this problem.

At the top, before the ball is dropped, it has all U_G , and right before it hits the floor the ball has all E_K , and since the total energy didn't change, we can set them equal to each other: $E_K = U_G$.

Therefore, $mgh = \frac{1}{2}mv^2$. Rearranging the equation, the Pokeball's speed is $v = \sqrt{2gh}$.

ELASTIC FORCE AND POTENTIAL ENERGY

In many problems on the AP exam, you'll see springs used in problems to add an extra element of knowledge. Springs are **elastics** - when compressed or elongated, they want to return to their original, relaxed state. The force that the string applies on whatever is pulling it is called the **restoring force**, or occasionally the spring force.

The formula for spring force is $F_S = -kx$, where k is the “spring constant” of the spring, and x is the distance stretched.

A spring constant measures the spring's resistance to being stretched (every spring is different). A spring with a higher spring constant will require more force to stretch the same distance as a spring

with a lower spring constant. Since $k = \frac{F}{x}$, the units for a spring constant are N/m (Newtons per meter)

The “distance stretched” is the extra distance the spring has compressed or stretched (in meters). If the spring is 5 cm long, and when stretched is 10 cm long, then the “x” value is 5 cm, or 0.05 meters.

Most of the time, you won’t need the negative in the formula, but that is how it is denoted on the formula sheet. This shows that the force is always trying to oppose the displacement vector of the spring. This negative is more important when we get to the Simple Harmonic Motion unit.

One important lab application to determine the spring constant of a spring. The easiest way to do this is just to orient the spring vertically, then hang a mass from it. After the spring is stable, you have a known force (gravity) and you can measure the stretched distance, so you can calculate the spring constant by setting the force of gravity equal to the spring force: $F_G = F_S$, and get the solution $mg = kx$.

When a spring is compressed, there is potential energy stored in it. This energy can be transferred to an object that the spring is contact with. Also, an object can come in contact with the spring and store energy in it.

If a spring is **ideal**, that means it can be governed by the $F=-kx$ equation, and that there is no energy lost when things collide with the spring. The energy stored in a spring (**elastic potential energy**) can be found with the formula: $U_S = \frac{1}{2}kx^2$. We can use this formula whenever we are dealing with problems that deal with conservation of energy.

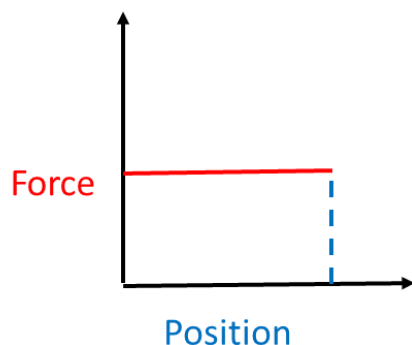
WORK

What is work? Work is, by definition, change in kinetic energy. Whenever an unbalanced external force is applied on a system, work is done on it.

On the formula sheet, work is written as

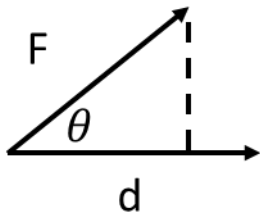
$$W = \int \mathbf{F} \cdot d\mathbf{r}$$

Which means it is the integral of force with respect to position (r).



A quick recall of basic calculus reminds us that an integral is the area under a curve. But, when the force is constant, we can solve for the area just by doing the force multiplied by the change in position. Usually, we are dealing with a constant, unbalanced force.

This means that usually we can just do $F \cdot d$. But, we need to make sure we ONLY use the component of force that is parallel to the displacement.



To get the component of force that is parallel to the displacement vector, we do $F \cos(\theta)$. If they are already parallel, the angle is 0, and cosine of 0 is 1.

Our complete formula is then

$$W = F \Delta x \cos(\Theta)$$

Work (Joules) Force (N) Displacement (m) Angle between force and displacement of object

This is also the **scalar product**, or **dot product** of the two vectors F and Δx , in which we care only about components parallel to each other.

Here's a basic practice problem with work.

An Ivysaur is pulled right with a tension force of 15 N, and a frictional force of 5 N opposes the motion. The box is displaced 2.5 m from its starting position. What is the work done by each force, and what is the net work on the object?

Solution: The tension force is in the same direction as the displacement, so we do $15N * 2.5m * \cos(0) = 37.5 J$

The frictional force is in the opposite direction as the displacement, so we do $5N * 2.5m * \cos(180) = -12.5J$

NOTE that if the force and displacement are in opposite directions, the force is doing negative work on the object. The net work can be calculated by adding these works together, resulting in 25J. Alternatively we could calculate the net force (10N) and do $10N * 2.5m * \cos(0) = 25 J$.

Here's another practice problem:

Joe holds a 50 kg Dragonite at a height of 1.5 m for 2 minutes. What is the work done by his applied force?

Solution: NO WORK IS DONE! There is no displacement at all in the direction of the applied force - Joe may be getting super tired, but he's actually doing no work. If displacement is 0, then work is 0!

WORK-ENERGY EQUIVALENCE

One of the most important formulas is $W = \Delta KE$. Work equals change in kinetic energy, and thus also equals the negative change in potential energy.

For example: If we drop a 2kg ball from a height of 10 meters, the work done by gravity is $W = Fd = (2kg) \left(9.8 \frac{N}{kg}\right) (10 m) = 196 J$. The change in potential energy is $\Delta U = mgh_f - mgh_i = -196 J$.

From this example, we can see that $W = -\Delta U = \Delta KE$

Another formula that comes as a result of this is $F = -\frac{dU}{dx}$, Force is the negative derivative of potential energy with respect to position. If force is constant, then the change in potential energy is also constant. This is mostly applied mathematically on the AP exam.

POWER

Power is known as the rate of energy transfer, and can be solved as the amount of work W done over a time interval t . Thus, the formula is $P = \frac{W}{t}$, and it is measured in Watts (J/s). Power can also be calculated as $P = Fv$, when the object is moving at a constant velocity pushed by a constant force.

If the power is changing (Work is changing non-linearly with respect to time) then instantaneous power can be found with $P = \frac{dW}{dt}$.

MOMENTUM

Momentum is a measure of how difficult it is to stop a moving object. The formula for momentum is $p = mv$, and it is measured in kg*m/s.

The law of conservation of momentum states that, if no external force is applied on the system, the sum of all individual momenta will be the same before and after a collision.

Objects in motion also have kinetic energy, **but kinetic energy is not conserved in collisions.** Although, if a collision is specified as elastic, then kinetic energy is indeed conserved.

In inelastic collisions (which are most collisions in the world) momentum is conserved, but kinetic energy is lost to thermal energy, meaning mechanical energy is not conserved.

Conservation of momentum can be demonstrated in a variety of ways:

$$m_1v_1 = m_2v_2$$

$$m_1v_1 + m_2v_2 = m_3v_3$$

$$m_1v_1 = m_2v_2 + m_3v_3$$

$$m_1v_1 + m_2v_2 = m_3v_3 + m_4v_4$$

The first situation is when there is no collision **in the dimension of motion of the original object**. This will be elaborated on later, but for now all you need to know is that the object's mass could change, leading its velocity to change as well because the momentum is conserved.

The second situation (top left) is when two objects collide and stick together to become one more massive object.

The third situation (bottom right) is when one object breaks up into two objects (usually in opposite directions) and momentum is conserved.

The fourth situation is when two objects collide and remain separate, either having directional changes, mass changes, or speed changes.

There are obviously other situations, but these are the common ways that conservation of momentum is used in solving problems. Let's look at a sample:

Problem: Sally (38 kg) and Phillip (43 kg) are on an ice rink, and can slide freely. Sally is holding a 5 kg pumpkin. Sally tosses the pumpkin with a speed of 5 m/s to Phillip, who is initially moving at 10 m/s in the same direction as the pumpkin. What is a) Sally's velocity after throwing the pumpkin? And b) Phillip's velocity after catching the pumpkin?

Solution: Firstly, we know they are on ice skates, so there is no friction and they can move freely.

So, first solving part a, we know that initially Sally and the pumpkin have a cumulative mass of 43 kg, and are stationary (0 m/s). Afterwards, the pumpkin is moving away from Sally at 5 m/s. Thus, we can form the following conservation of momentum equation.

$$(43 \text{ kg}) \left(\frac{0 \text{ m}}{\text{s}} \right) = (38 \text{ kg})(v_1) + (5 \text{ kg}) \left(5 \frac{\text{m}}{\text{s}} \right)$$

Since there was 0 total momentum before, there must be 0 total momentum after:

$$-(38 \text{ kg})(v_1) = 25 \text{ kg} \frac{\text{m}}{\text{s}}$$

$$v_1 = -0.66 \text{ m/s}$$

This means that Sally is moving **backward** at 0.66 m/s, which is what we'd expect: a slow speed rebounding away from the direction she threw the pumpkin (this is the same reason guns have recoil when bullets are fired).

As for Phillip, he already has a speed before, and the two objects (him and the Pumpkin) become one object. So we can write his conservation of momentum equation like so:

$$(43 \text{ kg}) \left(10 \frac{\text{m}}{\text{s}} \right) + (5 \text{ kg}) \left(5 \frac{\text{m}}{\text{s}} \right) = (48 \text{ kg})(v_3)$$

$$455 \text{ kg} \frac{\text{m}}{\text{s}} = (48 \text{ kg})(v_3)$$

$$v_3 = 9.48 \text{ m/s}$$

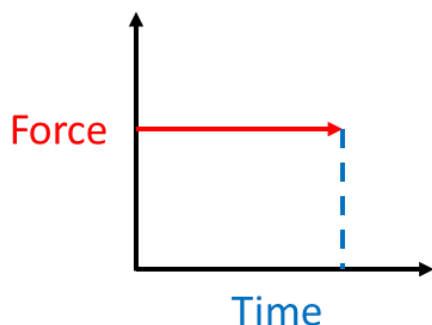
Since his mass increased slightly, he has slowed down slightly as well. This is an expected result.

MOMENTUM IN 2D

Things aren't always moving in straight lines, though. We must also deal with objects moving in two dimensions. When something is moving in both dimensions, we must divide the momentum up into x and y dimensions, and make sure each dimension is conserved after a collision. There won't be an example for this, but there will be one like this on the worksheet with a solution as well.

IMPULSE

Momentum is always conserved if there are no external forces on the system. But what if there were internal forces? Then the object or system has an impulse applied on it. Impulse, put simply is a **change in momentum**. It can also be expressed as the area under a force time graph. If the force is constant, the graph looks like:



In this case, Force multiplied by time gives us the area under the graph, giving us the general equation for impulse: $I = \Delta p = Ft$, where force is the constant force acting on an object and the time is the time that it acts on the object for. This is usually just a plug and chug in AP problems, but it is also important to know that the **impulse vector points the same direction as the force vector**.

COMBINING ENERGY AND MOMENTUM

For many problems you'll have to use energy and momentum together. For example, if something slides down a frictionless ramp and then collides with a different cart, we can use energy for the first half of the problem, solve for the velocity of the object, then use momentum for the collision part. These problem solving strategies are essential for success on the free response section of the AP test, and will be practiced in the worksheets.