Name Key

U5: Homework Questions #1

Rotational Kinematics and Moment of Inertia

1. A bar on a hinge starts from rest and rotates with an angular acceleration $\alpha = (14 + 5t)$ rad/s^2, where t is in seconds. Determine the angle in radians through which the bar turns in the first 4.09 s.

174 rad

$$\int 14+5t = 14t + \frac{5}{2}t^2$$

$$\int_0^{4.09} 14t + \frac{5}{2}t^2 = 7t^2 + \frac{5}{6}t^3$$

$$\rightarrow \boxed{174.11 \text{ rad}}$$

2. A centrifuge in a medical laboratory rotates at an angular speed of 3450 rev/min. When switched off, it rotates 48.0 times before coming to rest. Find the constant angular acceleration of the centrifuge.

-217 rad/s^2

$$0 = (361.28)^2 + 2\alpha(48 \cdot 2\pi)$$

$$3450 \frac{\text{rev}}{\text{min}}$$

$$(3450)(2\pi)\frac{\text{rad}}{\text{min}}$$

$$\frac{(3450)(2\pi)}{60}\frac{\text{rad}}{\text{s}}$$

$$\alpha = \boxed{-216.39 \frac{\text{rad}}{\text{s}^2}}$$

3. A rotating wheel requires 3.03 s to rotate through 37.0 revolutions. Its angular speed at the end of the 3.03 s interval is 97.9 rad/s. What is the constant angular acceleration of the wheel?

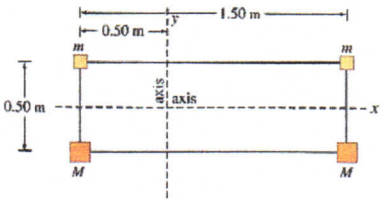
14 rad/s^2

$$\theta = \frac{1}{2}(\omega + \omega_0)t$$

$$37(2\pi)\text{rad} = \frac{1}{2}(97.9 + \omega_0)3.03$$

$$\omega_0 = 59.55 \frac{\text{rad}}{\text{s}}$$

$$\frac{97.9 - 55.55}{3.03} = \boxed{13.98 \frac{\text{rad}}{\text{s}^2}}$$



4. In the figure to the right: $m = 1.8$ kg and $M = 3.3$ kg.

(a) Calculate the moment of inertia of the array of point objects shown above about the vertical axis.

6.375 kg·m²

$m(.5m)^2 + M(0.5m)^2 + m(1m)^2 + M(1m^2)$

$(1.8kg)(.5m)^2 + (3.3kg)(0.5m)^2 + (1.8kg)(1m)^2 + (3.3)(1m)^2 = 6.375$

(b) Calculate the moment of inertia of the array of point objects shown above about the horizontal axis.

.6375 kg·m²

$2((1.8kg)(.25)^2) + 2((3.3kg)(.25)^2) = .6375$

5. A rigid rod of length L and mass M rotates about its center of mass (z axis in picture). Prove that the moment of inertia for the rod is:

$$I = (1/12)ML^2$$

$\int r^2 dm$

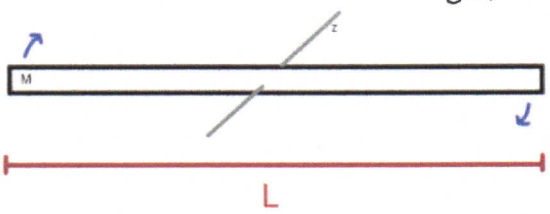
$\int r^2 \frac{M\,dr}{L}$

$\frac{M}{L}\int_{-1/2L}^{1/2L} r^2 dr$

$\frac{M}{L}\frac{r^3}{3}\Big|_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{1}{12}ML^2$

$\frac{dm}{M} = \frac{dr}{L}$

$dm = \frac{M\,dr}{L}$



6. A solid cylinder rotates about an axis that lies directly through its center. Prove that the moment of inertia of this cylinder I is equal to:

$$I = \tfrac{1}{2}MR^2$$

$\rho = \frac{M}{V} = \frac{dm}{dv}$

$dm = \rho\,dv$

$dv = L2\pi r\,dr$

Use these two substitutions and you should get it.

*A Hint:

*With the bar examples, we looked at the ratio dm/dr. Here, think about starting with a ratio of dm/dV and trying to get it back to dr.

- "V" is volume