Name <u>Key</u>

Unit 5: Worksheet #3

Rotational Energy and Angular Momentum

$I = mr^2$

1. A student sits on a freely rotating stool holding two weights, each of mass 3.04 kg. When his arms are extended horizontally, the weights are 0.93 m from the axis of rotation and he rotates with an angular speed of 0.757 rad/s. The moment of inertia of the student plus stool is 3.04 kg·m² and is assumed to be constant. The student pulls the weights inward horizontally to a position 0.301 m from the rotation axis.

$I_i = 3.04 + (3.04)(.93^2)(2) = 8.299 \text{ kgm}^2$

$I_f = 3.04 + (3.04)(.301^2)(2) = 3.591 \text{ kgm}^2$

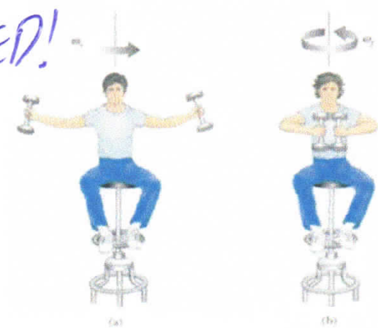
(a) Find the new angular speed of the student.

1.75 rad/s

MOMENTUM IS CONSERVED!

$(8.299)(.757) = (3.591)(\omega_f)$

$\boxed{\omega_f = 1.75 \frac{rad}{s}}$



(b) Find the kinetic energy of the rotating system before and after he pulls the weights inward.

2.38 J (before)

$\frac{1}{2}(8.299)(.757)^2 = 2.38 \text{ J}$

5.50 J (after)

$\frac{1}{2}(3.591)(1.75)^2 = 5.50 \text{ J}$

WOW! ENERGY IS NOT CONSERVED!

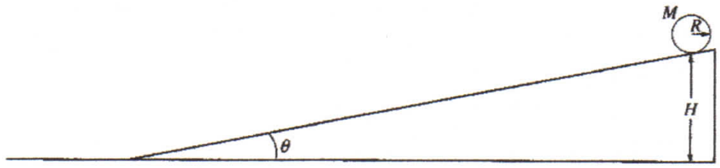
2. A playground merry-go-round of radius R = 1.60 m has a moment of inertia I = 255 kg·m² and is rotating at 9.0 rev/min about a frictionless vertical axle. Facing the axle, a 24.0 kg child hops onto the merry-go-round and manages to sit down on its edge. What is the new angular speed of the merry-go-round?

7.25 rev/min

$(255)(9) = (255 + (24)(1.6^2))(\omega_f)$

$\boxed{\omega_f = 7.25 \frac{rev}{min}}$

FUNNY HOW MOMENTUM IS ALWAYS CONSERVED!



THIS COMES UP A BUNCH

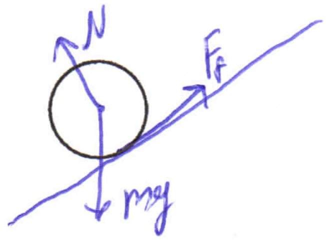
3. A solid cylinder with mass M, radius R, and rotational inertia ½MR² rolls without slipping down the inclined plane shown above. The cylinder starts from rest at a height H. The inclined plane makes an angle θ with the horizontal. Express all solutions in terms of M, R, H, θ, and g.

a. Determine the translational speed of the cylinder when it reaches the bottom of the inclined plane.

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right) \qquad gh = \frac{1}{2}v^2 + \frac{1}{4}v^2 = .75v^2$

$\boxed{v = \sqrt{\frac{gh}{.75}}}$

b. On the figure below, draw and label the forces acting on the cylinder as it rolls down the inclined plane Your arrow should begin at the **point of application** of each force.



$\boxed{a = \frac{2}{3}g\sin\theta}$

c. Show that the acceleration of the center of mass of the cylinder while it is rolling down the inclined plane is (2/3)g sinθ.

$mg\sin\theta - F_f = ma \qquad F_f r = \left(\frac{1}{2}mr^2\right)\frac{a}{r} \qquad F_f = \left(\frac{1}{2}m\right)(a) = \frac{1}{2}ma$ sub back in

d. Determine the minimum coefficient of friction between the cylinder and the inclined plane that is required for the cylinder to roll without slipping.

$\mu = \frac{F_f}{N} \qquad F_f = \frac{1}{2}ma \qquad N = mg\cos\theta$

$\mu = \frac{\frac{1}{2}ma}{mg\cos\theta} = \frac{\frac{1}{2}\left(\frac{2}{3}g\sin\theta\right)}{g\cos\theta} \qquad \boxed{\frac{1}{3}\tan\theta}$

e. The coefficient of friction μ is now made less than the value determined in part (d), so that the cylinder both rotates and slips.

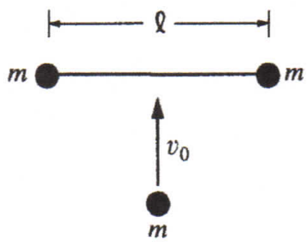
i. Indicate whether the translational speed of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the translational speed calculated in part (a). Justify your answer.

less energy will go into rotational energy

ii. Indicate whether the total kinetic energy of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the total kinetic energy for the previous case of rolling without slipping. Justify your answer.

Heat.

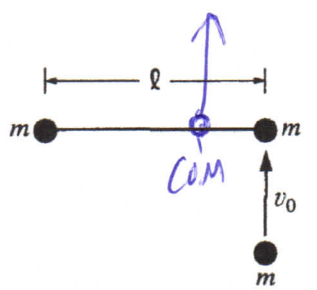
4. A space shuttle astronaut in a circular orbit around the Earth has an assembly consisting of two small dense spheres, each of mass m, whose centers are connected by a rigid rod of length *l* and negligible mass. The astronaut also has a device that will launch a small lump of clay of mass m at speed v_0. Express your answers in terms of m, v_0 *l*. and fundamental constants.



a. Initially, the assembly is "floating" freely at rest relative to the cabin, and the astronaut launches the clay lump so that it perpendicularly strikes and sticks to the midpoint of the rod, as shown above.

i. Determine the total kinetic energy of the system (assembly and clay lump) after the collision.

ii. Determine the change in kinetic energy as a result of the collision.



$MV_0 = (3m)(V_f)$ - Linear momentum

$V_f = \frac{\cancel{m}V_0}{3\cancel{m}} = \frac{V_0}{3}$

$KE_f = \frac{1}{2}(\cancel{3}m)\left(\frac{V_0^2}{\cancel{9}\,3}\right) = \boxed{\frac{V_0^2 m}{6}}$

$KE_i = \frac{1}{2}(m)(V_0^2) = \frac{V_0^2 m}{2}$

$\frac{V_0^2 m}{6} - \frac{3V_0^2 m}{6}$

$\boxed{-\frac{V_0^2 m}{3}}$

b. The assembly is brought to rest, the clay lump removed, and the experiment is repeated as shown above, with the clay lump striking perpendicular to the rod but this time sticking to one of the spheres of the assembly.

i. Determine the distance from the left end of the rod to the center of mass of the system (assembly and clay lump) immediately after the collision. (Assume that the radii of the spheres and clay lump are much smaller than the separation of the spheres.) $\frac{2m\ell}{3m} = \boxed{\frac{2\ell}{3}}$ ~~it moved~~ - new COM

ii. On the figure above, indicate the direction of the motion of the center of mass immediately after the collision. From above $\frac{V_0}{3}$

iii. Determine the speed of the center of mass immediately after the collision.

iv. Determine the angular speed of the system (assembly and clay lump) immediately after the collision.

$\left((m)(\tfrac{1}{3}\ell)^2\right)\left(\frac{V_0}{\ell/3}\right) = \left((m)(\tfrac{2}{3}\ell)^2 + (2m)(\tfrac{\ell}{3})^2\right)(\omega_f)$

$\frac{\cancel{3}\cancel{\ell^2} m V_0}{\cancel{9}\cancel{\ell}\ 3} = \frac{6m\ell^2}{9}(\omega_f)$

$\boxed{\omega_f = \frac{V_0}{2\ell}}$