Name Key

Test Prep Worksheet

Simple Harmonic Motion

1. A mass is attached to a spring on a frictionless, horizontal surface. When it's set into oscillation, its period is T. An equal mass collides head-on with this mass, and the two masses stick together. The oscillation period is now:
   a. T
   b. √(2)*T
   c. 2T
   d. T/√(2)
   e. T/2

$T = 2\pi\sqrt{m/k}$

$\sqrt{\frac{2m}{k}}$

$(\sqrt{2})(2\pi)(\sqrt{\frac{m}{k}})$

Questions 2-4

A 2kg mass oscillates vertically at the end of a spring according to:

$$x(t) = 4\sin\left(\frac{\pi}{6}t + \frac{\pi}{8}\right)$$

2. The period of oscillation is:
   a. 3s
   b. (1/3)s
   c. (1/12)s
   d. 12s
   e. (π/6)s

$T = \frac{2\pi}{\omega}$ $\quad T = \frac{2\pi}{\pi/6}$

$\omega = \frac{\pi}{6}$ $\quad T = 12s$

3. The spring constant has a value in N/m of:
   a. $8\pi^2 / 9$
   b. $72\pi^2$
   c. $1{,}152\pi^2$
   d. $\pi^2 / 18$
   e. $81 / 8\pi^2$

$T = 2\pi\sqrt{m/k}$

$k = 4\pi^2 \sqrt{m/T^2}$

$T = 12s$

$m = 2kg$

$k = \frac{\pi^2}{18}$

4. The maximum kinetic energy of the mass is:
   a. 16 J
   b. (2π/3) J
   c. 4 J
   d. 32 J
   e. $(4\pi^2/9)$ J

$V(t) = \frac{dx}{dt}$

$V(t) = \frac{4\pi}{6}\cos\left(\frac{\pi}{6}t + \frac{\pi}{8}\right)$

max of cos = 1

$V = \frac{4\pi}{6}$

$KE = \frac{1}{2}(2)\left(\frac{2\pi}{3}\right)^2$

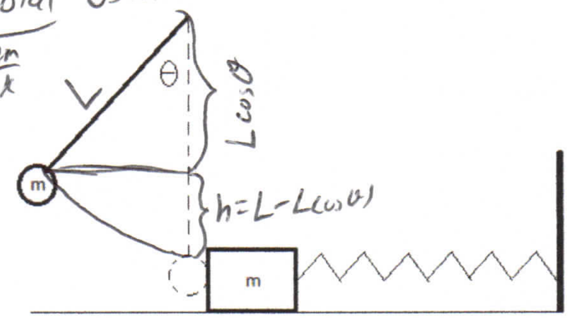
$= \frac{4\pi^2}{9} J$

5. A mass *m* is attached to a light string of length *L*, making a simple pendulum. It is displaced an angle θ from the vertical and released at t = 0. Directly below the pivot of the pendulum is a stationary second mass *m* equal to the first, attached to a spring of constant *k* on a frictionless, horizontal surface. When the first mass collides with the stationary mass, the first mass detaches from the string and sticks to the second mass.
   a. At what time will the spring first reach its maximum compression?
   b. Find the amplitude of the spring oscillations

a. Time to reach total compression is 1/4 total oscillation

$t = \frac{1}{4}T_p + \frac{1}{4}T_{sp}$ $\quad T_p = 2\pi\sqrt{\frac{L}{g}}$ $\quad T_{sp} = \sqrt{\frac{2m}{k}}$

$t = \frac{\pi}{2}\left(\sqrt{L/g} + \sqrt{2m/k}\right)$



b. $U_g = E_k$ $\quad mv = (2m)(v_f)$

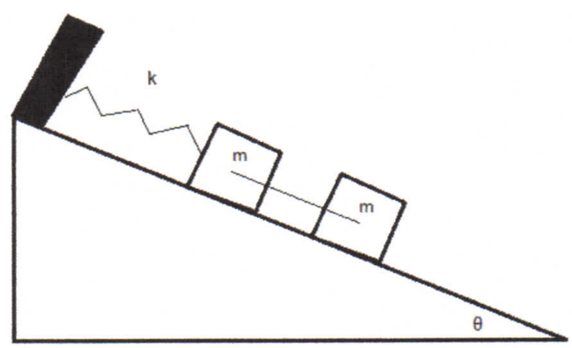
$v = \sqrt{2gL(1-\cos\theta)}$ $\quad V_f = \frac{1}{2}\sqrt{2gL(1-\cos\theta)}$ $\quad \frac{1}{2}(2m)(v_f)^2 = \frac{1}{2}kA^2$ $\quad A = \frac{mgL(1-\cos\theta)}{K}$

6. Two equal mass *m* connected by a light string are currently at rest on a frictionless surface inclined at an angle θ. One of the masses is connected by a spring with constant *k* to a point at the top of the incline. At t = 0, the string is cut, and the mass connected to the spring begins to oscillate.
   a. Determine the period of the oscillations
   b. Determine the amplitude of the oscillations
   c. In terms of the given quantities, write an expression for the velocity of the oscillating mass at an arbitrary time

a. Gravity doesn't change period $T = 2\pi\sqrt{m/k}$

b. F_k, F_N, F_g $\quad F_k = F_g \sin\theta$

$kx = 2mg\sin\theta$



$x_1 = \frac{2mg\sin\theta}{k}$ – before

$x_2 = \frac{mg\sin\theta}{k}$ – after

$A = x_1 - x_2 = \frac{mg\sin\theta}{K}$

c. $x(t) = A\cos(\omega t + \phi)$

$v(t) = -\omega A\sin(\omega t + \phi)$

$v(t) = \sqrt{m/k}\,(g\sin\theta)\sin\left(\sqrt{\frac{K}{m}}t\right)$