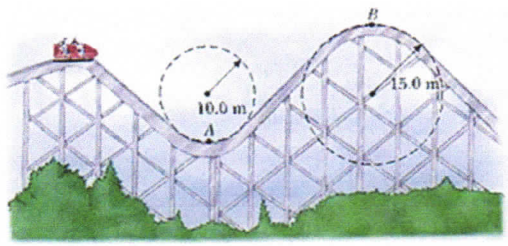
Name Llave

Worksheet #2

Circular Motion and Universal Gravitation

1. A roller-coaster car has a mass of 498 kg when fully loaded with passengers.



(a) If the car has a speed of 20.2 m/s at point *A*, what is the force exerted by the track on the car at this point?

25,200 N

$F_c = F_N - mg$

$\frac{mv^2}{r} = F_N - mg$

$F_N = \frac{mv^2}{r} + mg$

$\frac{(498\,kg)(20.2\,m/s)^2}{(10\,m)} + (498\,kg)(9.8\,N/kg) = \boxed{25{,}200\,N}$

(b) What is the maximum speed the car can have at *B* and still remain on the track?

12.1 m/s

$F_N \approx$ really small number, we can assume it equals 0.

$\frac{mv^2}{r} = mg$

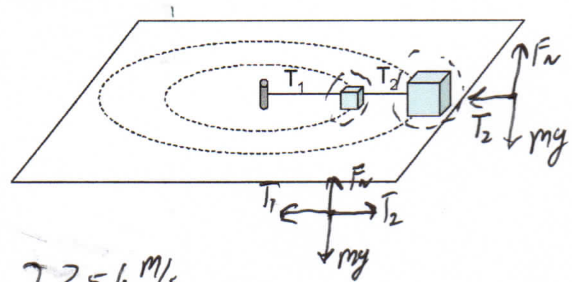
$(v^2) = (15\,m)(9.8\,N/kg)$

$\boxed{v = 12.1\,m/s}$

2. Two masses, 30 g and 50 g, are attached in line via two strings and spun around in uniform circular motion on a frictionless horizontal surface, as shown below. The radius of the 30 g mass's motion is 0.5 m and the radius of the 50 g mass's motion is 0.75 m. Determine the tension in the inner string (T_1) and in the outer string (T_2) when the masses are spun around 5 times every 10 seconds.

T_1 0.52 N

T_2 0.37 N



5 times per 10 s

1/2 time per second

$T = 2\,s \quad f = 1/2$

small block

$v = \frac{(2)(\pi)(0.5\,m)}{2\,s}$

$v = 1.571\,m/s$

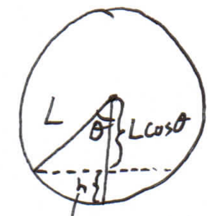
big block

$v = \frac{2(\pi)(0.75\,m)}{(2\,s)} = 2.356\,m/s$

$\frac{(0.05)(2.356)^2}{0.75} = \boxed{T_2 = 0.37\,N}$

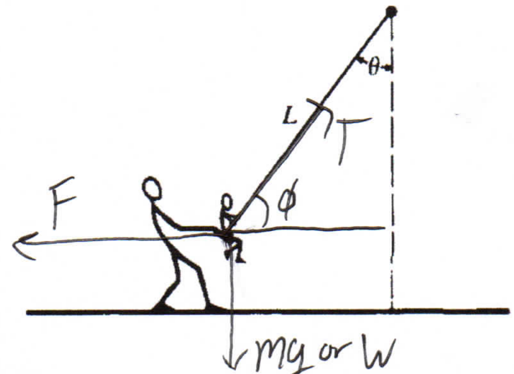
$\frac{(0.03)(1.571)^2}{0.5} = T_1 - T_2 \qquad \boxed{T_1 = 0.52\,N}$

Geometry lesson:



we need to find h

$h = L - L\cos\theta$



*Actual AP Problem

3. An adult exerts a horizontal force on a swing that is suspended by a rope of length L, holding it at an angle θ with the vertical. The child in the swing has a weight W and dimensions that are negligible compared to L. The weights of the rope and of the seat are negligible. In terms of W and θ, determine

a. the tension in the rope;

$W = T_y$ T θ W $\cos\theta = \frac{W}{T}$ $\boxed{T = \frac{W}{\cos\theta}}$

b. the horizontal force exerted by the adult.

$F = T\sin\theta$

$F = \left(\frac{W}{\cos\theta}\right)(\sin\theta) = \boxed{W\tan\theta}$

c. the tension in the rope just after the release (the swing is instantaneously at rest);

T

mg = W

$T = W\cos\theta$

d. the tension in the rope as the swing passes through its lowest point.

Use Energy:

$mgh = \frac{1}{2}mv^2$

$g(L - L\cos\theta) = \frac{1}{2}v^2$

$v^2 = 2gL(1-\cos\theta)$

T W $F_c = T - W$

$\frac{m2g\cancel{L}(1-\cos\theta)}{\cancel{L}} = T - W$

$2W(1-\cos\theta) = T - W$

$T = 2W - 2W\cos\theta + W = \boxed{3W - 2W\cos\theta}$

$r_e = 6371 km$

4. You are explaining to friends why astronauts feel weightless orbiting in the space shuttle, and they respond that they thought gravity was just a lot weaker up there. Convince them and yourself that it isn't so by calculating how much weaker gravity is at $h = 220$ km above the Earth's surface.

0.935 $(g_h / g_{surface})$

$\frac{GMm}{r^2}$ $\quad \frac{\frac{\cancel{GMm}}{(r+220)^2}}{\frac{\cancel{GMm}}{r^2}} = \frac{r^2}{r^2+440r+220^2}$ $\boxed{\neq 0.934}$

seconds in 23.9 days

5. Plaskett's binary system consists of two stars that revolve in a circular orbit about a center of mass midway between them. This means that the masses of the two stars are equal. If the orbital velocity of each star is 220 km/s and the orbital period of each is 23.9 days, find the mass M of each star. (For comparison, the mass of our Sun is 1.99 $\times 10^{30}$ kg.)

105 solar masses

$V = \frac{2\pi r}{2064960}$

$(220 \times 10^3) = \frac{2\pi r}{2064960}$

$r = 7.23 \times 10^{10} m$

$F_c = F_g$

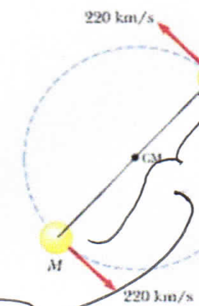
$\frac{\cancel{M}(220 \times 10^3 m/s)^2}{7.23 \times 10^{10}} = \frac{GM}{(2r)^2}$

same r we solved for

notice I'm using 2

$M = 2.097 \times 10^{32} kg$

divide by mass of sun to get $\boxed{105.34}$



*Actual AP Problem *(No Answers Given)*

6. An explorer plans a mission to place a satellite into a circular orbit around the planet Jupiter, which has mass $M_J = 1.90 \times 10^{27}$ kg and radius $R_J = 7.14 \times 10^7$ m.

a. If the radius of the planned orbit is R, use Newton's laws to show each of the following:

The orbital speed of the planned satellite is given by $v = \sqrt{\frac{GM_J}{R}}$	~~$\frac{mv^2}{R_J+R} = \frac{GM_Jm}{(R_J+R)^2}$~~ $\quad \frac{\cancel{m}v^2}{\cancel{R}} = \frac{GM_J\cancel{m}}{R^{\cancel{2}}}$ $\quad v^2 = \frac{GM_J}{R}$ $\quad v = \sqrt{\frac{GM_J}{R}}$
The period of the orbit is given by $T = \sqrt{\frac{4\pi^2R^3}{GM_J}}$	$\frac{2\pi R}{T} = \sqrt{\frac{GM_J}{R}}$ $\quad \frac{4\pi R^2}{T^2} = \frac{GM_J}{R}$ $\quad 4\pi R^3 = GM_JT^2$ $\quad T^2 = \frac{4\pi R^3}{GM_J}$ $\quad \boxed{T = \sqrt{\frac{4\pi R^3}{GM_J}}}$

b. The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55×10^4 s. Determine the required orbital radius in meters.

$3.55 \times 10^4 s = \sqrt{\frac{4\pi R^3}{(\underset{G}{6.67408 \times 10^{-11}})(\underset{M_J}{1.9 \times 10^{27} kg})}}$ $\quad \boxed{R = 233415997.1 m}$